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i) There is an emerging story about MPQM and QFT based partly on views inherited from physics, and partly on new work, especially that of Paul Teller and Michael Redhead. Three important steps in the tale go like this - (a) The transition from CM to QM involves significant change in particle metaphysics, (b) QFT cannot be viewed as particulate even in a MPQM sense, although (c) an ontologically enlightening understanding of QFT can be obtained from the occupation representation. Though there is some truth to all of these claims, I wish to argue that, as usually understood, they are in fact false.

The ever present danger in metaphysical debates such as these is that participants will be running around on the same pitch but playing different games; some waving cricket bats at the ball, some aiming for a netball hoop and others trying for a touch-down. In the hopes that we can all play together, or at very least so that others can understand the game which I am playing, I want to briefly describe how I think interpretation of a physical theory can be profitably carried out.

What I propose is that the metaphysical implications of a theory be investigated by analysis of its formal models; that is by demonstrating what properties they have or are consistent with. I am particularly concerned with the spaces of possible states, and with the general structure, independent of specific dynamical systems. What follows is a survey of various philosophically interesting properties that are the consequences of the kinematics of CM, MPQM and QFT. Even if you do not find my suggestion

ultimately satisfying, it has the virtue that it gives my metaphysical claims clear meaning - they describe the space of states. To see more clearly how such a programme is to work, I will take up the first point by way of illustration.

ii) It is a common claim that statistical mechanics reveals an important metaphysical difference between the classical and quantum worlds (see for instance, Dieks 1990, Lavine 1991, Redhead and Teller 1992, Reichenbach 1956, ter Haar 1964). Though arguments vary, the underlying intuition is that in many particle systems, say a pair of coins, classical statistics reveal four possibilities (Heads-Heads, Heads-Tails, Tails-Heads and Tails-Tails), but quantum statistics allow only three (two-heads, one-heads-one-tails and two-tails). On this view classical permutations can produce distinct states, and, intuitively, one can identify a single coin as having different properties in qualitatively identical worlds. Thus, commentators suggest, classical coins have a more robust metaphysic of identity than do quantum coins. The two flavours of statistics are often distinguished as Maxwell-Boltzmann and Bose-Einstein respectively, but since I will show that both possibilities are empirically adequate in the classical world, we should avoid such glib use.

The notion of identity at play here needs to be carefully clarified; in the first place to distinguish it from, say, numerical identity, and in the second so that we have a well-defined doctrine to discuss. Fortunately David Lewis (1986)

provides us with the notion of 'haecceitism', a property of modal universes of possible worlds such as a space of 'possible states'. To say that the four state universe is haecceitistic means no more or less than this: there is a world, qualitatively identical to the world 'heads-tails', in which the heads-up coin is instead tails-up. In other words, individuals have counter-parts determined in non-qualitative ways. On the other hand, if there is only one heads-tails state the property cannot hold, and so the three-state universe is not haecceitistic. It should be apparent that studying this property realises my advertised method of interpretation; haecceitism is a property of a universe of possible worlds, and a state space is exactly that. The claim under investigation is that CM entails haecceitism and that QM rules it out: elsewhere (Huggett 1994) I discuss the latter suggestion, here I wish to debunk the former.

In this simple case there is an empirical difference between one-heads-one-tails appearing one time in two and appearing one time in three, but such a description is unrealistic, since real coins have many more degrees of freedom than heads-or-tails. A realistic classical system for our purposes is a genuine gas of atoms with independent canonical positions and momenta. It is for atoms in just such a system that it is supposed that classical identity dissolves in QM; and it is for just such a system that we can see that the supposed classical identity is not an empirical matter.

The generalisations of the four and three state coin state-

spaces are the full and reduced phase spaces - equivalently the phase and distribution spaces. For n particles each with m degrees of freedom a state is represented in phase space as a point in an n.m dimensional cartesian space. Clearly permutations of single particle positions and momenta yield rotations to distinct points, and hence one can argue that distinct states differ only in the way that properties are distributed over atoms; in phase space, atoms have identity in the sense under discussion. If we couple this argument with the claim that phase space (when equipped with an indifferent measure) is required to explain observable thermodynamical properties, then we have an empirical argument for haecceitism.

In the reduced phase space any two points related by a permutation are identified as representing the same state - equivalently, states are represented as a distribution over single particle states. Explicitly, states are of the form $(O_1,...O_c)$, where O_i is equal to the number of atoms in the single particle state labelled 'i'. Such a reduced phase can be called a distribution. It is clear that there are no distinct reduced states that differ only by a rearrangement of properties amongst the atoms, and atoms correctly represented in such a way do not have the appropriate flavour of identity. Since it is phase space counting that we are all familiar with in statistical mechanics, and from reflection on toy systems like the coins it is easy to conclude that the reduced phase space is in some way inappropriate for explaining the phenomena of classical thermodynamics, and that

it only when quantum phenomena make themselves felt that such a representation becomes useful. It is equally easy to see that such a conclusion is badly wrong-headed, for there can be no difference in the statistical properties of the two systems.

As is very familiar, we can use binomial coefficients to see that the ratio of distinct phases to any distribution of n atoms and c distinct single particle states is;

$$N_\Gamma(O_1,\ldots O_c):N_Z(O_1,\ldots O_c) = n!/_{i=1}\pi^c O_i!:1. \qquad \text{eqn 1}$$

Empirical differences between the two representations is supposed to be manifested in statistical properties, ideally, in the frequency with which given distributions occur. Given the difference in counting and assuming identical measures it certainly appears that the two spaces will exhibit such potentially observable differences. For instance, in the coin world there are two distinct heads-tail phases, so that distribution appears one time in two. We shall grant that indeed the data of classical statistical mechanics as inferred from thermodynamics reveals that the correct frequencies are obtained by indifference over phase space. What we shall disprove is the further claim that the indifference in the reduced phase space, with its weaker metaphysical structure fails to predict the same data - the two spaces are, in the relevant cases, empirically equivalent.

The point is simple; in classical gases we can make the assumption that all atoms are in distinct individual states, so

$O_i \leq 1$. The quickest way to see that this is true is to point out that in a continuum, particle co-incidences are vanishingly unlikely. (And one can make this more rigourous, by deriving statistical properties, for instance the Maxwell-Boltzmann equilibrium distribution $n_{MB}(q) \propto \exp -E_q/kT$, directly from the fact that state-cells in distribution space have vanishing volume). On this assumption on 'exclusion' we see;

$$N_\Gamma(O_1,..O_c):N_Z(O_1,..O_c) = n!:1,$$

inserting 0!=1!=1 into eqn 1. Of course, for there are n! distinct permutations of n distinct one-particle states. Hence if there are Z $(=\{n+c-1\}!/n!\{c-1\}!)$ distinct distributions, there are n!Z phases, and the frequency of a given distribution, derived from indifference, in the two representations is;

$$F_\Gamma(O_1,..O_c) = n!/n!Z = 1/Z = F_Z(O_1,..O_c).$$

Of course, since the ratio of distributions to phases is a constant, n! is an irrelevant constant that washes out in frequencies.

Let me emphasise the significance of this result. There are no physical properties that depend on how properties are arranged over particular atoms, at most, properties supervene on the precise distribution of classical continuum canonical co-ordinates. In the case of a classical gas we are interested in the frequencies with

which distribution dependent properties occur, and the maximal information we could in principle obtain is thus the frequency with which a precise distribution occurs. A moments reflection shows that the frequency with which states of co-incidence occur is zero, and that of another moment that the frequency of any remaining distribution is the same in both the full and reduced phase spaces. Thus, in realistic physical situations, the state spaces have identical statistical consequences for distributions, and hence since properties at most supervene on reduced phases, observation can in no way select one over the other.

In particular, if the continuum spaces are coarse grained into blocks so that many atoms can be in a single block, the distribution over the blocks is a function of the distribution over the (near) continuum. The frequency of any block distribution is thus derived from the frequencies of the corresponding continuum distributions, and thus by the above result must be the same whichever way we count continuum distributions. This of course means that statistical physics is indeed captured by either space. The result I've given shows this must be true, but it can be simply proved directly as well (Huggett 1994).

There is then nothing here to settle any issue about the metaphysics of atoms, and hence the discussion of quantum mechanics which emphasises the claim that 'identity' evaporates in transition is misguided. Put another way, using the full and reduced phase spaces to explain the difference between Maxwell-Boltzmann and Bose-Einstein counting is inept; the difference between classical

and quantum does not lie here. 'Maxwell-Boltzmann' should be reserved for any representation that predicts the correct equilibrium distribution law - as both our spaces do. This is not to say that there are no differences worth mentioning between classical and quantum particles, but I will leave that discussion for another place.

iii) The second part of the story we are investigating is the claim that there is another important revolution in the transition from MPQM to QFT. This is for instance assumed in Redhead and Teller (1992); the authors argue that the tensor product formalism is poisoned by a classical notion of identity, 'LTI', and should be replaced by a 'Fock space description, free of labels and LTI' (p217). In other words, we need a description without particles. Clearly what they have in mind is the mode excitation representation,

$$|n_1, n_2, \ldots\rangle = (n_1! n_2! \ldots)^{1/2} (a_1^*)^{n1} (a_2^*)^{n2} \ldots |0\rangle,$$

in which reference is only made to the occupation level of some single particle basis.

No one denies that the QFT formalism, with indistinct particle number presents new difficulties for atomists, but it is terribly misleading to point to the absence of particle labels as the source of these problems. It is to assume that simply because we don't give a proper name to something, it therefore cannot exist! A much

more fruitful way of looking at the question is to again consider the way the world is represented through the state space of the system. If we can show, labels not withstanding, equivalence with a particle representation, then we will have shown the possibility of a particle interpretation.

The relevant result is that an n-particle subspace of, say, symmetrised Fock space is equivalent to the symmetric sector of the n-particle tensor product space; with corresponding mappings of relevant operators. This is a familiar result in non-relativistic QM, see eg Greenberg and Raboy (1981), but I am unaware of any demonstrations in the relativistic domain. In the first place there is no interacting relativistic MPQM, and so physically there is less interest in showing the equivalence. In the second there is a difficulty in constructing a many particle relativistic Schrodinger equation even for the free case. As noted by Smith and Weingard (1987), the Einstein equation, $E^2=p^2+m^2$, and the postulate that the Hamiltonian is the generator of time translations, $H=i\partial/\partial t$, suggests the following free equations of motion;

Single particle: $-(\partial/\partial t)^2|\phi_i\rangle = (p_i^2 + m^2)|\phi_i\rangle$

Two particle: $-(\partial/\partial t)^2|\phi\rangle = (p_1^2 + p_2^2 + 2m^2)|\phi\rangle$,

where $|\phi_i\rangle$ (i=1,2) are single particle states, and $|\phi\rangle$ is a two particle linear superposition of products of single particle states, $|\phi_1\rangle|\phi_2\rangle$. Unfortunately, because these equations are quadratic in time, the product form of the joint state is

inconsistent with $|\phi\rangle$ and $|\phi_i\rangle$ being solutions of their respective equations of motion.

There is however a way around these problems if one opts to work in the momentum representation,

$$\phi_i(p,t) = \langle p|\phi_i(t)\rangle,$$

for this basis is, of course, diagonal in energy,

$$H\phi_i(p,t) = (p^2 + m^2)^{1/2}\phi_i(p,t) \equiv w_p\phi_i(p,t).$$

Thus our desired, linear, single particle equation of motion is, in this representation,

$$i\partial_t\phi_i(p,t) = H\phi_i(p,t) = w_p\phi_i(p,t).$$

This equation of motion can be used to define a relativistic MPQM, and hence to set-up the desired particle-field equivalence. If we now follow the usual path and demand that the n particle states be symmetrised products of single particle states,

$$\phi(p_1,..p_n,t) = \Sigma_{perms}\phi_1(p_1,t)...\phi_n(p_n,t),$$

we can find a consistent Schrodinger equation, by taking the free hamiltonian to be the sum of single particle hamiltonians acting on the various one-particle states;

$$H_n = H\otimes I\otimes...I + I\otimes H\otimes I\otimes...I + ... I\otimes I\otimes...H,$$

which yields,

$$i\partial_t\phi(p_1,..p_n,t) = H_n\phi(p_1,..p_n,t) = {}_{i=1}\Sigma^n w_{pi}\phi(p_1,..p_n,t). \quad \text{eqn 2.}$$

Note that this representation is logically compatible with the claim that the system is comprised of n individuals, each associated with a single particle subspace. As you can probably see, this claim will run into trouble with the principle of the identity of indiscernibles for symmetrised states, but we no longer take that as a first principle of natural philosophy. The individuation would seem to have to be a brute metaphysical fact, and an ugly one at that, but the mere form of MPQM does not rule out individuals.

So, I urge, if the QFT Fock representation is equivalent to the many particle formalism, then it too is consistent with individuals, whether we express states with or without labels. And such an equivalence is easily obtained, for quantisation of the Klein-Gordon field (see Schweber, 1964 for details) reveals that a suitable basis for representing the canonical commutation algebra is,

$$\{|0\rangle, |k_1,...k_n\rangle: n=1,2..\},$$

in which modes are created or destroyed by the familiar creation

and annihilation operators, a^*_k and a_k. eg,

$$|k_1,..k_n\rangle = (n!)^{-1/2}a^*_{k1}...a^*_{kn}|0\rangle.$$

Continuing to offer mere highlights of the treatment, the quantised scalar field stress-energy tensor can be expressed as a 'weighted' number operator,

$$H_{KG} = \int d^3k/w_k . w_k a^*_k a_k .$$

Application to an arbitrary state, $|k_1,...k_n\rangle$ yields,

$$H_{KG}|k_1,...k_n\rangle = {}_{i=1}\Sigma^n w_{ki} . |k_1,...k_n\rangle$$

Thus the physical interpretation is that this is a basis of energy eigen states, with the usual understanding about definite values that goes with it. This of course is exactly the same interpretation given to the n-particle relativistic theory that I developed earlier; and it immediately follows on the assumption that the energy generates time translations that the QFT also has the same equations of motion (eqn 2) as that theory.

Thus, if we map n particle momentum wavefunctions into field wavefunctions in n-excitation subspaces, the observables and equations of motion agree. Thus, not only is there a structural equivalence between the representations, they also have the same physical interpretations. It is because of this 'equivalence' that

QFT can purport to be a complete theory even in a world that is manifestly particulate. Its formal structure contains elements that are exactly those of a particle theory. Of course, QFT also allows superpositions across n-particle sub-spaces so the equivalence is not full, even if we take MPQM to be the set of all n-particle theories. However, this issue is logically posterior to the question of 'labels' emphasised by commentators such as Redhead and Teller.

Let me spell this out; if it is the switch from labels to excitation level talk which is metaphysically significant then there should be no particles even in the case of free field states of definite number. But in just that case the field state space is the same as that of MPQM, and that is consistent with individuals. The representation of QFT has a structure compatible with atomism, and that remains true however you decide to express the states. This result does not mean that we have to accept atomism at this level, but rather that the choice is not forced upon us by the theory itself. If we reject atoms at this level it must be on extra-theoretical grounds.

iv) Finally I want to register my doubts about the value of an interpretation of QFT based upon the occupation representation; a strategy advocated in Teller (1990). These reservations are described in detail in a paper "Interpretations of QFT" (Huggett and Weingard, 1994) by Robert Weingard and me in the last issue of Philosophy of Science, and I will not repeat them here. Rather I

want to explain how they fit into the general pattern of argument that I have been describing here.

The three steps that I have been criticising today do not so much constitute an interpretation in themselves, but rather a series of claims about what significant issues need to be accounted for in an interpretation. First, it is supposed that a salient fact about the connection between classical and quantum mechanics is the evaporation of phase space individuality. But this itself can hardly be significant if classical physics holds no commitment to the full phase space. Second, it is supposed that a salient fact about the connection between MPQM and QFT is that since the latter has no particle labels, there is no notion of individuals. But in the relevant way, MPQM and QFT are equivalent, so the issue of labels is a red herring. And finally, Bob and I argue that, attractive as the occupation representation may seem, it simply is not the most significant way of looking at QFT.

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Block the reduced phase space into b blocks of p (near) continuum cells. Maxwell-Boltzmann counting means that there are N_{MB} states corresponding to a distribution $(n_1,\ldots n_b)$, of n atoms.

$$N_{MB}(n_1,\ldots n_b) = n!/_{i=1}\Pi^b n_i!$$

From the reduced underlying phase space, we assume c>>n. Hence it follows that

$N_Z(n_1,\ldots n_b)$ = ways of picking n_1 cells out of p. x... ways of picking n_b cells out of p.

$$= p!/n_1!(p-n_1)!\ x\ p!/n_2!(p-n_2)!\ x\ \ldots p!/n_b!(p-n_b)!$$

$$= {}_{i=1}\Pi^b\ p!/n_i!(p-n_i)!$$

But for d>>e, $d!/(e-d)! \sim d^e$, so

$$\sim {}_{i=1}\Pi^b\ p^{ni}/n_i!$$

$$= p^n/_{i=1}\Pi^b\ n_i!$$

$$= p^n N_{MB}(n_1,\ldots n_b)/n!.$$

Thus the counting differs only by a distribution independent constant, and hence the frequencies agree.